THE APPLICATION OF NUMERICAL RELATIVITY TO LIGO AND LISA DATA ANALYSIS

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Recent advancements in the field of numerical relativity now make it possible to utilize the previously-unmodeled merger segment of a binary black hole (BBH) inspiral, merger, and ringdown in the search for and characterization of gravitational wave signals. The implications for LIGO and LISA include an enhanced signal-to-noise ratio for all BBH events that merge in-band due to increased contribution of the merger signal, which increases the event rate and improves parameter estimation, and a means to test different theories of gravity by comparing measured signals to simulations in the strong field regime of the merger. We present some preliminary expectations of the impact of these results on gravitational wave data analysis.

It is well-documented that if a known signal is embedded in noise, then matched filtering is the optimal mechanism for extraction of that signal. The signal to noise r that can be achieved by matched filtering for a signal $h(t)$ embedded in noise $n(t)$ with a one-sided power spectral density of $S_h(f)$ is given by

$$r^2 = \left(\frac{<h|h>}{rms<h|n>}\right)^2 = 4\int_0^\infty df\,\frac{|\tilde{h}(f)|^2}{S_n(f)}.$$

For an imperfect template, r is instead proportional to the inner-product of the template with the actual underlying signal, so r improves as the template becomes a better match to the signal. The primary purpose of numerical relativity, at least from the perspective of data analysis, is to provide the most accurate and precise template families possible, both to maximize the measurable r and to extract parameter values for source characterization once a signal has been identified.

For the purpose of data visualization, it is convenient to plot the noise background as sky- and inclination- averaged rms strain, given by $<(f\,S_h(f))^{1/2}>_{\text{sky,inc.}}$, since this term is independent of integration time (assuming stationarity of the noise). Furthermore, if the signal template is plotted as polarization-averaged characteristic strain, given by $<2f\,|h(f)|>_{\text{pol.}}$, then the height of the signal above the noise times the frequency band of the signal is equal to r. Figures 1 and 2 show these data for specific modeled sources of LIGO and LISA, respectively. Of particular note is the "wiggle" in the characteristic signal strain which results from eccentricity of the orbit in the numerical simulation due to inaccuracy in the initial data. As the simulation runs, the orbit circularizes, and the strain flattens out.

The accumulated signal-to-noise is a particularly useful statistic for determining the relative contributions of different phases of the waveform evolution to the overall signal-to-noise. Its value at a given frequency is simply r integrated from zero to that frequency, rather than from zero to infinity. The colored circles indicate the designated times before the peak gravitational wave amplitude is reached, and the black boxes indicate, from low frequency to high, the accumulated signal-to-noise at the traditional merger and ringdown onset values (i.e. from Flanagan and Hughes (1997)) and the total signal-to-noise for the entire waveform.

A matched filter template that is initially in phase with a signal buried in noise will contribute to the integrated SNR as the template ramps up in frequency until the accumulated phase error of the template reaches $\pi/2$, at which point the local inner product goes to zero, and so the contribution to the signal-to-noise becomes zero. An optimized template will have maximized phase overlap with a signal in the band where the characteristic strain of the signal maximally exceeds the rms strain of the noise. To be optimal for the majority of cases, a generalized template should have minimal phase error near the merger (where the local signal amplitude is by far the largest) and minimal phase error for the broadest range of frequencies possible. To accomplish this, Post Newtonian (PN) approximations will be used to model the inspiral phase of the waveform until the accumulated phase error reaches $\pi/2$. From the high-frequency end, results from numerical relativity simulations will be used for as broad a band as they can validly cover. If the phase, which is arbitrary to within a constant, is fixed at the merger, then a PN approximation can also be used to extend the low frequency boundary of the numerical simulations. Since the phase is known to within the accuracy of the simulation at this low frequency boundary, a PN approximation can be used to extrapolate to still-lower frequency until, once again, the accumulated phase error reaches a value of $\pi/2$. Figure 3 represents the bandwidth range that can contribute to signal-to-noise given the current low-frequency boundary of numerical simulations. PN correction terms are shown with all terms set to be zero either at $f = 0$ ($t \to -\infty$) or at $f = 0.07$ (in dimensionless units), which is the current numerical low-frequency boundary. This assumes that the phasing correction of the 3.5 PN order term sets an upper limit to the phase error that results from higher-order corrections, which appears to be reasonable given the behavioral trend of lower PN orders, which are also shown.

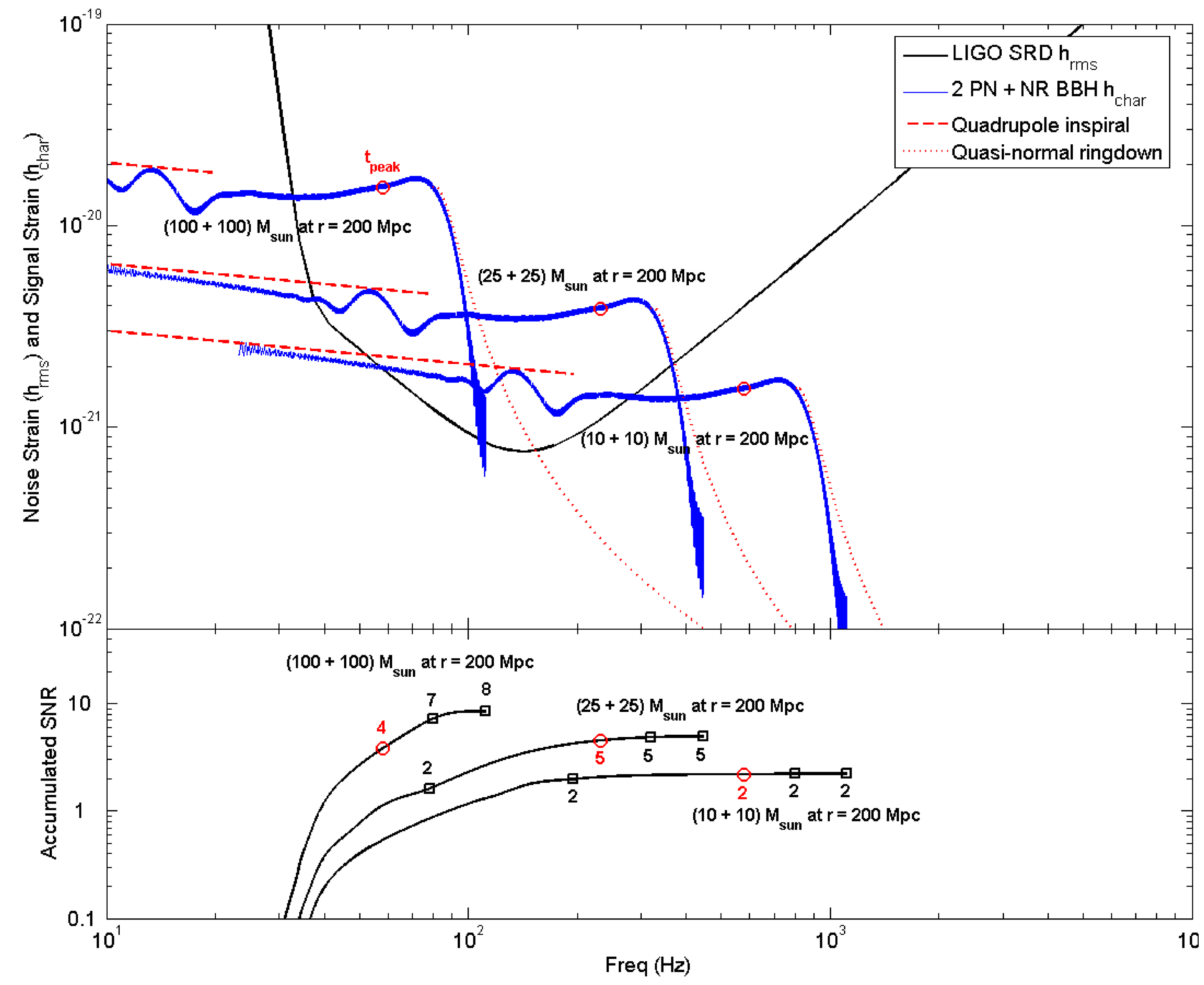

Figure 1. LIGO Noise h_{rms} and the h_{char} of a 2 PN inspiral tied to an NR merger and ringdown, with the corresponding accumulated SNR.

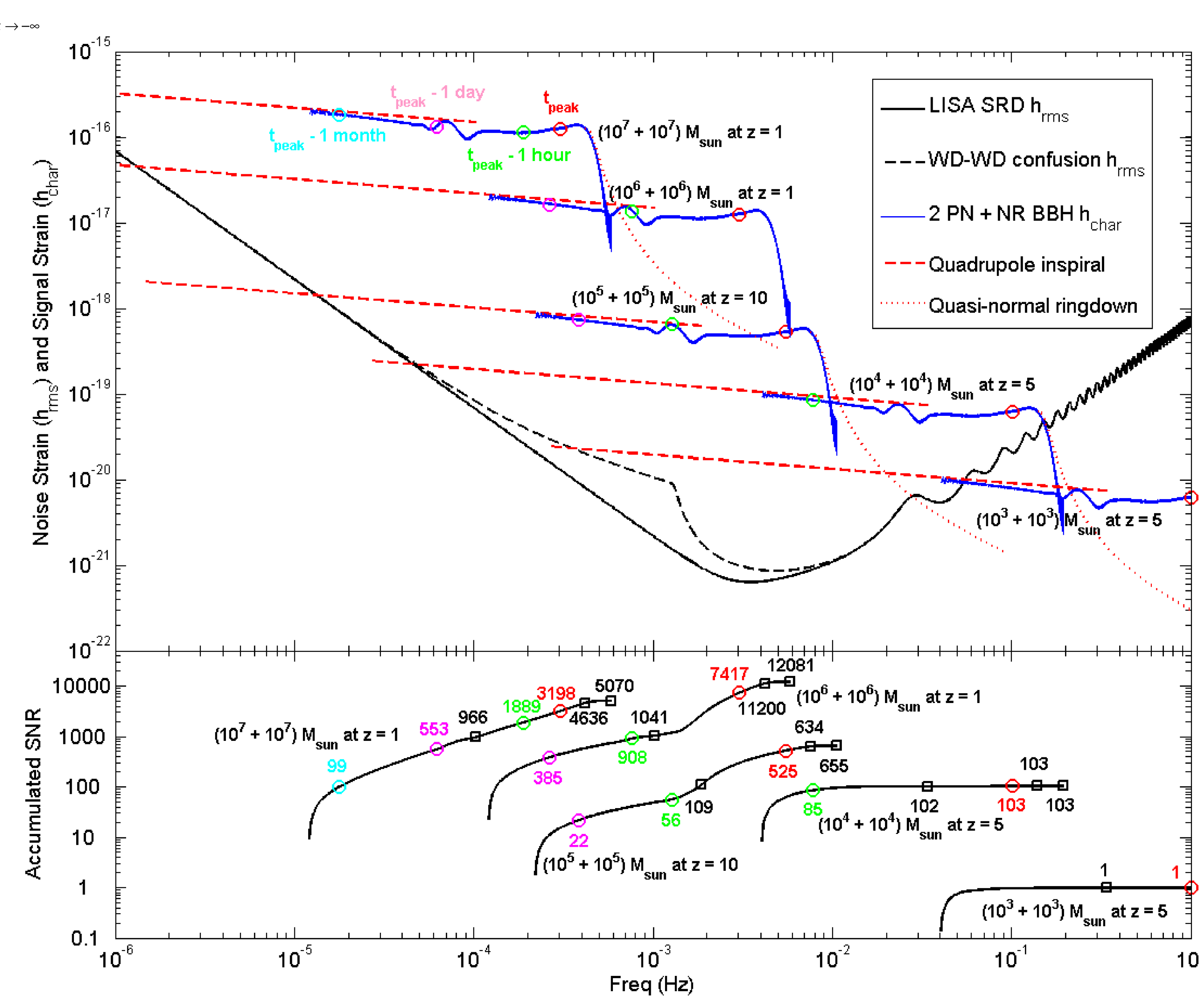

Figure 2. LISA Noise h_{rms} and the h_{char} of our signal, with accumulated SNR.

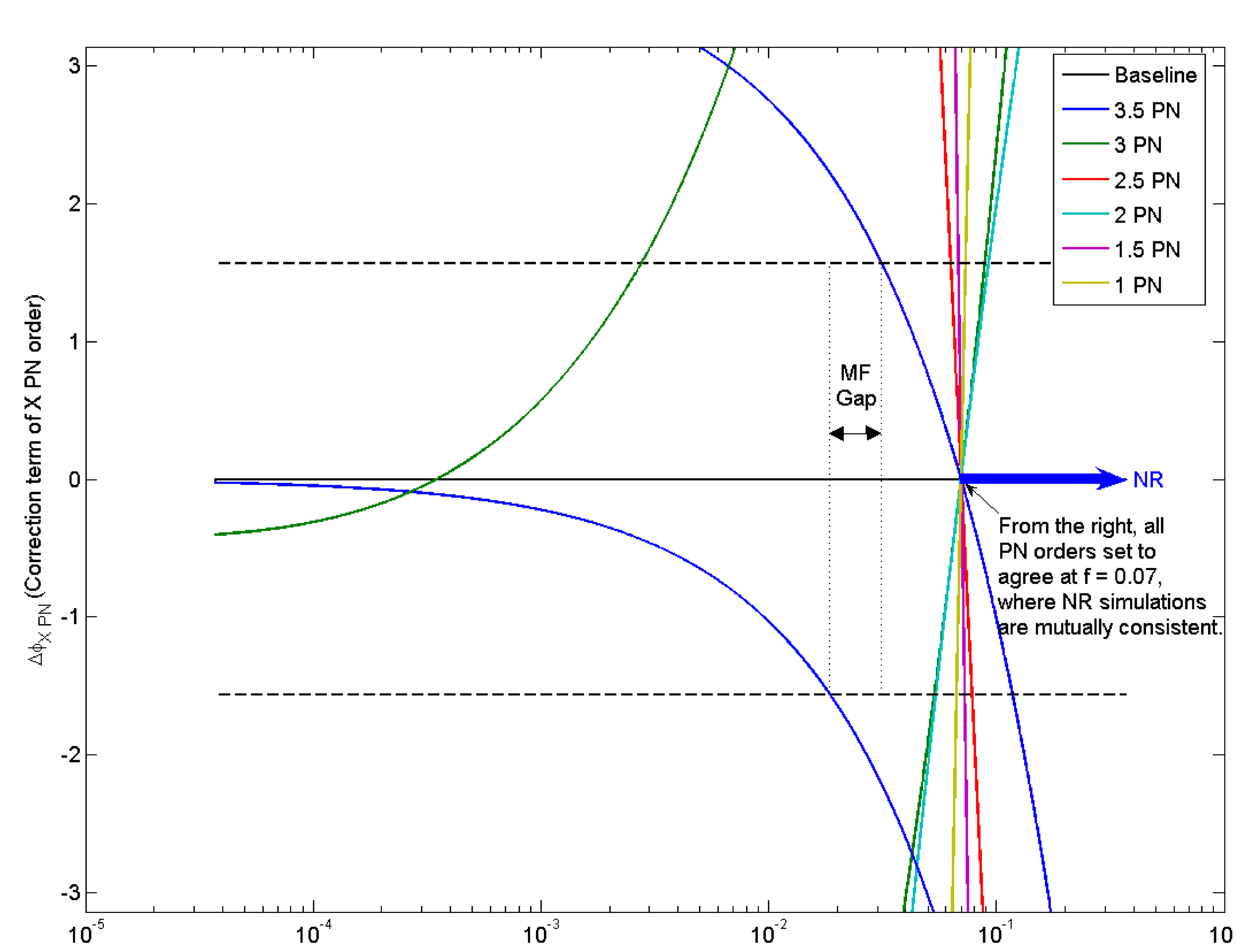

Figure 3. Accumulated phase error from all current PN order corrections.